

2. The function f is defined by

$$f : x \mapsto 2x, \quad x \in \mathbb{R}.$$

(a) Find $f^{-1}(x)$ and state the domain of f^{-1} .

(2)

The function g is defined by

$$g : x \mapsto 3x^2 + 2, \quad x \in \mathbb{R}.$$

(b) Find $gf^{-1}(x)$.

(2)

(c) State the range of $gf^{-1}(x)$.

(1)

5. (a) Using the formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

$$(i) \quad \sin (A + B) - \sin (A - B) = 2 \cos A \sin B, \tag{2}$$

$$(ii) \quad \cos (A - B) - \cos (A + B) = 2 \sin A \sin B. \tag{2}$$

(b) Use the above results to show that

$$\frac{\sin(A + B) - \sin(A - B)}{\cos(A - B) - \cos(A + B)} = \cot A. \tag{3}$$

Using the result of part (b) and the exact values of $\sin 60^\circ$ and $\cos 60^\circ$,

(c) find an exact value for $\cot 75^\circ$ in its simplest form. (4)

6.

Figure 1

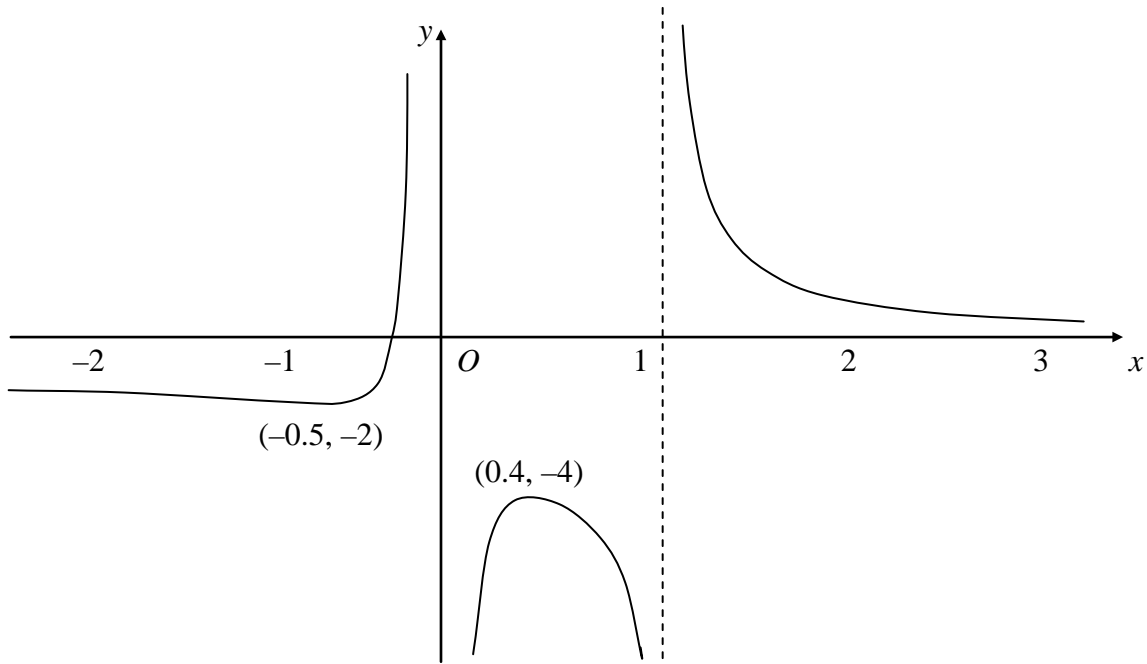


Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a minimum point at $(-0.5, -2)$ and a maximum point at $(0.4, -4)$. The lines $x = 1$, the x -axis and the y -axis are asymptotes of the curve, as shown in Fig. 1.

On a separate diagram sketch the graphs of

- (a) $y = |f(x)|$, (4)
- (b) $y = f(x - 3)$, (4)
- (c) $y = f(|x|)$. (4)

In each case show clearly

- (i) the coordinates of any points at which the curve has a maximum or minimum point,
- (ii) how the curve approaches the asymptotes of the curve.

7. (a) Sketch the curve with equation $y = \ln x$. (2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point $(e, 1)$ passes through the origin. (3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = e$ if $0 < m < \frac{1}{e}$. (2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = e^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5 , giving your answer to x_5 to 3 decimal places. (3)

The root of $\ln x - \frac{1}{3}x = 0$ is α .

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α , correct to 3 decimal places. (3)

